

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/23
Paper 2 Pure Mathematics 2 (P2)
May/June 2012
1 hour 15 minutes

Additional Materials: | Answer Booklet/Paper |
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| Graph Paper |
| List of Formulae (MF9) |

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Solve the equation $\left|x^{3}-14\right|=13$, showing all your working.

2


The variables $x$ and $y$ satisfy the equation $y=A\left(b^{x}\right)$, where $A$ and $b$ are constants. The graph of $\ln y$ against $x$ is a straight line passing through the points $(0,2.14)$ and $(5,4.49)$, as shown in the diagram. Find the values of $A$ and $b$, correct to 1 decimal place.

3 The polynomial $\mathrm{p}(x)$ is defined by

$$
\mathrm{p}(x)=a x^{3}-3 x^{2}-5 x+a+4
$$

where $a$ is a constant.
(i) Given that $(x-2)$ is a factor of $\mathrm{p}(x)$, find the value of $a$.
(ii) When $a$ has this value,
(a) factorise $\mathrm{p}(x)$ completely,
(b) find the remainder when $\mathrm{p}(x)$ is divided by $(x+1)$.

4 (i) Given that $35+\sec ^{2} \theta=12 \tan \theta$, find the value of $\tan \theta$.
(ii) Hence, showing the use of an appropriate formula in each case, find the exact value of
(a) $\tan \left(\theta-45^{\circ}\right)$,
(b) $\tan 2 \theta$.


The diagram shows the curve $y=4 \mathrm{e}^{\frac{1}{2} x}-6 x+3$ and its minimum point $M$.
(i) Show that the $x$-coordinate of $M$ can be written in the form $\ln a$, where the value of $a$ is to be stated.
(ii) Find the exact value of the area of the region enclosed by the curve and the lines $x=0, x=2$ and $y=0$.

6 A curve has parametric equations

$$
x=\frac{1}{(2 t+1)^{2}}, \quad y=\sqrt{ }(t+2)
$$

The point $P$ on the curve has parameter $p$ and it is given that the gradient of the curve at $P$ is -1 .
(i) Show that $p=(p+2)^{\frac{1}{6}}-\frac{1}{2}$.
(ii) Use an iterative process based on the equation in part (i) to find the value of $p$ correct to 3 decimal places. Use a starting value of 0.7 and show the result of each iteration to 5 decimal places.

7 (i) Show that $(2 \sin x+\cos x)^{2}$ can be written in the form $\frac{5}{2}+2 \sin 2 x-\frac{3}{2} \cos 2 x$.
(ii) Hence find the exact value of $\int_{0}^{\frac{1}{4} \pi}(2 \sin x+\cos x)^{2} \mathrm{~d} x$.

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